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13. ABSTRACT *(Maximum 200 words)*

In this study, three related areas were investigated with the objective of developing a continuum model for streamflow synthesis. The areas spanned surface runoff to subsurface unsaturated flow to flood-wave propagation to comparative assessment of different dynamic wave representations of the shallow water-wave theory.

The surface runoff was modeled using discrete linear models for overland flow, and physically-based Muskingum methods for channel flow. Also developed was a theory of errors for a comparative evaluation of the kinematic, diffusion, and dynamic wave representations of the shallow water-wave theory.

The unsaturated flow was modeled using a systems-based model for infiltration and soil moisture, and the kinematic-wave theory for movement of soil moisture. The infiltration model was a general one encompassing the popular models of Horton, Kostiakov, Overton, Green and Ampt, and Philip as special cases.

The models developed in the project were verified using field data. In most cases, simplified models were found to be adequate.

14. SUBJECT TERMS
Continuum model, error analysis, infiltration models, shallow water-wave theory, soil moisture movement, streamflow synthesis, subsurface unsaturated flow, systems approach, surface runoff.

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A CONTINUUM MODEL FOR STREAMFLOW SYNTHESIS

COMPLETION REPORT

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1. INTRODUCTION

Streamflow evolves as a continuum, and is normally comprised of three components: (1) surface runoff, (2) interflow, and (3) baseflow. These components occur concurrently, although their relative magnitudes vary with time. If we consider a sudden burst of rainfall, then surface runoff predominates during the rising part of the streamflow hydrograph, interflow predominates during the early part of its recession, and baseflow predominates during the delayed part of its recession. The mechanisms and, therefore, the governing equations, of these components are different but are influenced by dynamic interactions prevailing between them.

Although streamflow synthesis has long been a subject of scientific inquiry, treatment of streamflow as a continuum taking into account dynamic interactions amongst its components has not yet been fully developed. Most of the approaches of streamflow synthesis are based on the concepts embodied in Horton's infiltration theory of runoff (Horton, 1933). According to this theory, rainfall is absorbed for intensities not exceeding infiltration capacity, while for excess rainfall there is a constant rate of absorption as long as the infiltration capacity is unchanged. Thus, infiltration divides rainfall into two parts. One part travels over the surface giving rise to surface runoff, and the other part infiltrates into the ground resulting in replenishment of soil moisture and recharge of groundwater, and eventually in interflow and baseflow.

The three components of streamflow have been treated at various levels of mathematical sophistication but in virtual isolation with one another. Surface flow has been studied for over half a century (Woolhiser, 1982), and as a result, it is understood reasonably well (Hall, 1982). Likewise, baseflow contribution to streamflow has been studied for nearly 30 years and it too is understood reasonably well (Hall, 1982). The same, of course, cannot be said about interflow. This is not even well defined and is least understood. Also, least understood are the dynamic interactions prevailing amongst these components.

Although considerable progress has been made in mathematical and numerical treatment of the boundary value problems dealing with flows over impervious beds, the understanding of surface flows over porous beds which dynamically interact with subsurface flow is quite limited. The importance of this interaction has been pointed out in the past in the context of border irrigation (Parlange, 1973), and in the study of flood waves in ephemeral streams (Smith, 1972). These studies, however, are not based upon a coupled set of equations pertaining to surface and subsurface flow; rather the attenuation in surface flows is included by considering certain infiltration rate with time lag. The dynamic diffusion due to infiltration, therefore, remains unaccounted for.

Freeze (1972) was probably the first to develop a comprehensive quantitative treatment of hillslope hydrology considering explicit interactions between near-surface groundwater flow, surface runoff and rainfall intensity patterns. Rather limited work has since been done along the lines of Freeze. Some notable

examples are the conceptual model of Beven and Kirby (1979), the model of Hillel and Hornberger (1979), and the finite element model of Beven (1977).

The most recent work representing a major step forward in developing an analytical treatment of interdependent surface and subsurface hydrologic processes is by Smith and Hebbert (1983). In their model, the hillslope was considered to consist of two soil layers with the lower soil capable of restricting vertical flow at the interface creating a perched aquifer and subsurface stormflow. Unsaturated vertical flow was routed by a kinematic wave method and linked with an analytical infiltration model. Thus, this model attempted to integrate most of the major hydrologic response mechanisms presently identified as contributing to the hydrology of a simple hillslope. Other hillslope hydrological models (Cundy, et al., 1985; Stagnitti, et al., 1986) and surface irrigation models (Walker and Humpherys, 1983; Stagnitti, et al., 1986) and surface irrigation models (Walker and Humpherys, 1983; Ram, et al., 1983) have also been developed. However, none of these models developed a method to compute infiltration rate dynamically, although it is one of the major factors affecting runoff (or advance front) and surface water profile. Most of the models utilized empirical formulae such as Kostiakov's or Green and Ampt's, etc. Therefore, the prevailing dynamic process between surface and subsurface flows remained unaccounted for.

Toward the goal of eventually accomplishing a continuum model for streamflow synthesis, three related areas were investigated: (1) subsurface unsaturated flow, (2) flood wave propagation, and (3) comparative assessment of

different dynamic wave representations of shallow water wave theory. In particular, we focused on (a) comparative evaluation of the kinematic-wave, diffusion-wave, and dynamic-wave representations of the shallow water wave theory ubiquitously applied to modeling surface runoff, (b) development of a systems- based model for infiltration, and (c) modeling movement of soil moisture.

2. SURFACE RUNOFF MODELING

The surface runoff hydrology was investigated along three lines: (a) development of a theory of errors for comparative assessment of three shallow water-wave representations: (i) kinematic-wave, (ii) diffusion-wave, and (iii) dynamic-wave; (b) development of discrete linear models for watershed runoff; and (c) physically-based Muskingum methods of channel-flow routing. A short discussion of each is in order.

2.1 Theory of Errors

A wide range of problems involving free-surface flows can be modeled using the shallow water-wave (SWW) theory. The SWW theory is described by the St. Venant equations or their approximations. The three most popular representations of the SWW theory are the kinematic-wave (KW), diffusion-wave (DW), and the dynamic-wave (DYW) approximations.

One of the fundamental questions to be addressed in physically-based modeling of watershed runoff is one of determining the appropriate approximation of the shallow water-wave (SWW) theory. Of the different approximations of the SWW theory, the two most popular approximations are the kinematic-wave (KW) theory and the diffusion-wave (DW) theory. How accurate are these approximations? Which approximation should be used and under what conditions? What is the spatial or temporal distribution of error of a given approximation? What is the criterion to choose between these approximations? The past studies have dealt with development of criteria for judging the adequacy of these approximations. However, these criteria are point values and do not relate to errors resulting from use of these approximations. Consequently, the error in space and/or time is not known.

The larger goal of this study was to develop a theory of errors for quantitative evaluation of the adequacy of these approximations, and, in turn, of the shallow-water-wave theory. However, because the SWW theory consists of a system of nonlinear partial differential equations of hyperbolic type, derivation of error equations is unattainable at this stage. Therefore, some realistic simplifications were made. The first was the simplification of flows being time-independent. Steady state flows are ubiquitous in nature, and much of the early hydraulics was based on this assumption.

Because spatially distributed data are seldom available, it was assumed that the full form of the SWW theory or the dynamic-wave representation was the true representation or model, and was capable of mimicking the behavior of the real

world, prototype system, and the kinematic-wave and diffusion-wave approximations were reasonable approximations, but were germane to conceptual error. The adequacy of these approximations is well documented in hydraulic literature. However, what is not known is the actual error and its distribution in time and/or space, as well as its relationship to flow characteristics, system properties and initial and boundary conditions.

The theory of errors can serve as an objective criterion for judging the adequacy of the KW and DW approximations by comparison with the DYW approximation. For time-independent flows, the theory yields error as a function of space involving infiltration, and boundary conditions. The error differential equation is ordinary in place of partial, and is more amenable to numerical solution. Even with this simplification, analytical solutions are not possible but the numerical solutions are much simpler and easy to graph.

Different criteria have been developed to evaluate the adequacy of the KW and DW theories, but no explicit relations either in time or in space between these criteria and the errors resulting from these approximations have yet been derived. Furthermore, when synthesizing streamflow, it is not clear if the kinematic-wave and the diffusion-wave approximations are valid, on one hand, for the entire hydrograph or for a portion thereof, and on the other hand, for the entire channel reach or for a portion thereof. To put differently, all of these criteria take on fixed point-values for a given rainfall-runoff event. This study, under simplified conditions, derived error equations for the kinematic-wave and diffusion-wave approximations for space-

independent as well as for time-independent flows. These equations provided a continuous description of error in the streamflow hydrograph.

With these considerations in mind, errors of kinematic-wave and diffusion wave approximations were derived for steady-state channel flows subject to finite flow at the upstream end. The diffusion-wave approximation was in excellent agreement with the dynamic wave representation for a range of the values of the Froude number and the kinematic-wave number. The kinematic-wave approximation was also in good agreement with the dynamic wave representation, but for a limited range of the values of the Froude number and the kinematic-wave number. On the other hand, the approximate diffusion-wave analogy, although leading to analytical solutions, was not accurate and should not be employed.

Under two different initial conditions and two boundary conditions, solutions of the kinematic-wave and diffusion-wave equations were derived under the simplification that the flow was temporally independent. Thereafter, error equations for the KW and DW theories were derived. It was found that the DW theory was quite accurate and for Froude number (Fo) less than 2 and the kinematic wave number (K) greater than 10, the DW theory would be an accurate representation of the SWW theory. Under the condition where there was no downstream control, the KW theory was an accurate representation for $K > 30$, $K F_0^2 > 5$. The KW theory does not accommodate a downstream control and hence, as expected, did not accurately represent the SWW theory for the entire channel under any of the two boundary conditions. Details of this work are contained in Singh, Aravamuthan and Joseph (1994).

For space-independent flows, a dimensionless parameter was defined, reflecting the effect of initial depth of flow, channel-bed slope, lateral inflow, acceleration due to gravity, and channel roughness. For time-independent flows, the dimensionless parameter was the product of the kinematic-wave number and the square of the Froude number. By comparing the kinematic-wave and diffusion wave solutions with the dynamic-wave solution, error equations were derived in terms of the aforementioned dimensionless parameters. The error equations for space-independent flows turned out to have the form of the Riccati equation. The work is described in Singh, Aravamuthan and Joseph (1993).

2.2 Watershed Runoff

Discrete linear models were developed for estimating runoff and sediment discharge hydrographs from agricultural watersheds. A regression equation was also established relating runoff rate and sediment discharge. Tested on five small basins, the results were in good agreement with observations. For the discrete linear transfer runoff model, the values of the integral square error (ISE) were generally less than 1% for all calibration events, and around 10% with the average value of 9.36% for all verification events. For the discrete linear transfer sediment model, the calibration coefficient of determination R for all five basins was more than 97%, and the verification R was more than 91% with an average of 94.3%. Details of this work are described in Wang et al. (1991).

2.3. Physically - Based Muskingum Method

Flow routing in channels was investigated using the kinematic wave theory as well as the Muskingum method. For the latter method parameters were derived from the St. Venant equations. Preliminary testing showed that this method of parameter estimation made the Muskingum method more accurate than any of the conventional methods. The kinematic wave method was investigated for perennial as well as ephemeral streams. This method can be extended to include flood wave propagation due to dam rupture. This work is more fully described in Wang and Singh (1992).

3. SUBSURFACE FLOW

Modeling of flow of water in the unsaturated zone is far from complete, especially at the field scale. Two lines of inquiry were, therefore, launched. First, a systems approach was developed to model infiltration and soil moisture, which holds promise for unifying different infiltration models reported in the literature. This approach can also relate parameters of one infiltration to another. The second type of approach pertained to application of the kinematic wave theory. This approach has the advantage of coupling plant root extraction and evapotranspiration with soil water dynamics.

3.1 Infiltration Modeling

A general infiltration model was derived using systems approach. The models of Horton, Kostiakov, Overton, Green and Ampt, and Philip are some of the

example models which are shown as special cases of the general model. An equivalence between the Green-Ampt model and the Philip two-term model was shown. The general model also provides a solution for the Holtan model expressing infiltration as a function of time. This solution of the Holtan model does not appear to have been reported in the literature. A first-order analysis was performed to quantify the uncertainty involved with the generalized model. The general infiltration model contains five parameters. Two of the parameters are physically based and can therefore be estimated from the knowledge of soil properties, antecedent soil moisture conditions, and infiltration measurements. The remaining three parameters can be determined using the least squares method. The model was verified using ten observed infiltration data sets. Agreement between observed and computed infiltration was quite good. This work is more fully described in Singh and Yu (1990).

3.2 Movement of Soil Moisture

The unsaturated subsurface flow serves as a link between surface runoff and groundwater runoff, and, therefore, occupies the central position in the streamflow continuum. The unsaturated flow region is the upper soil matrix which also is the principal source of interflow. For surface flow, infiltration is the major sink, and for groundwater flow, soil moisture percolation is the principal source or recharge. Much of the mathematical treatment of unsaturated flow, reported to date, is based on the Fokker-Planck equation or Richards equation. Both these equations are of the diffusion type, and do not lend themselves to analytical solutions, except for overly simplified cases.

If one ignores the effect of diffusion and models soil moisture movement using the kinematics wave theory, then, under certain simplifying but realistic conditions, it is possible to derive analytical solutions. This premise was pursued in this project. Currently available one-dimensional kinematic-wave models assume absence of sink terms. In other words, once the water gets infiltrated, it is either retained by the soil or moves downward to recharge the groundwater. This assumption is not tenable, especially in agricultural or forest watersheds. In this project, an effort was made to include a sink term in modeling of soil moisture. This sink term may represent removal of soil moisture by vegetation through the process of evapotraspiration.

Recognizing the difficulties of modeling unsaturated flow using the Richards equation, a novel approach was developed in this project. This approach was based on the kinematic wave theory in which a unique relation is hypothesized between the flux and the flow concentration or the hydraulic head.

The fundamental assumption underlying this theory is that the moisture movement is gravity-dominated and the hydraulic conductivity-soil moisture relationship is single-valued, i.e., it does not experience any hysteretic effects. Although these assumptions are not strictly valid, they do provide a reasonably accurate approximation. Another advantage of the theory is its simplicity and that it allows analytical solutions under simplified conditions. Under more complex conditions, numerical solutions are easily derived.

This unique relation, when coupled with the continuity equation expressing the conservation of mass, gives rise to a first order, nonlinear hyperbolic equation. Under simplified initial and boundary conditions, analytical solutions of this kinematic-wave equation are tractable. Following this tract, the soil moisture movement was modeled with the use of the kinematic-wave theory. However, attention is to be focused on certain aspects of the kinematic-wave modeling that are not apparent at the first glance. First, because the time history of the moisture front distinguishing between wet and dry soil is unknown, the kinematic-wave formulation of soil moisture movement results in a free-boundary problem. Second, natural watersheds have vegetation either seasonally or throughout the year. Inclusion of vegetation in modeling of soil-moisture movement gives rise to an additional free boundary, further complicating the model.

With these considerations in mind, a kinematic-wave model was developed for simulating the movement of soil moisture in unsaturated soils with plants. The model involved three free boundaries. Analytical solutions were derived when the plant-root extraction of moisture was at a constant rate, and the upstream boundary condition was time independent. If these assumptions are waived, then numerical solution is the only resort.

With the use of this theory, a comprehensive analytical treatment has been developed for soil moisture movement with plant-root extraction. The treatment involves free boundaries and to our knowledge this has not been dealt with in the literature so far. This work is described in Singh and Joseph (1994).

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